# Boyut Çözümleme-Dimensional Analysis

| Fiziksel nicelik | İngilizce | Simge | Dimension |
|---|---|---|---|
| Uzunluk | Length | L | L |
| Kütle | Mass | M | M |
| Zaman | Time | T | T |
| Alan | Area | A | $L^2$ |
| Oylum | Volume | V | $L^3$ |
| Hız | Velocity | $v$ | L / T |
| İvme | Acceleration | $A$ | $L / T^2$ |
| Doğrusal momentum | Linear momentum | $mv$ | ML / T |
| Açısal momentum | Angular momentum | $m\Omega r$ | ML / T |
| Kütle yoğunluğu | Mass density | $\rho$ | $M / L^3$ |
| Sayı yoğunluğu | Number density | N | $1/L^3$ |
| Açısal hız | Angular velocity | $\Omega$ | 1/T |
| Kinetik erke | Kinetic energy | $mv^2/2$ | $ML^2 / T^2$ |
| Elektrik alan | Electric field | E | $M^{1/2} / L^{1/2} T$ |
| Manyetik alan yeğ. | Magnetic field int. | H | $M^{1/2} / L^{1/2} T$ |
| Frekans | Frequency | $\omega, \nu, \Omega$ | 1 / T |
| Basınç | Pressure | P | $M / LT^2$ |
| İş | Work | W | $M L^2 / T^2$ |
| Direnç | Resistance | R | T/L |
| Elektrik yükü | Electrical charge | q | $M^{1/2} L^{3/2} / T$ |

$$\frac{\partial}{\partial t} \rightarrow \frac{1}{T}$$

$$\nabla \Phi = \frac{\partial \Phi}{\partial x}\boldsymbol{i} + \frac{\partial \Phi}{\partial x}\boldsymbol{j} + \frac{\partial \Phi}{\partial x}\boldsymbol{k}$$

$$\nabla \cdot \boldsymbol{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}$$

$$\nabla \times A = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \partial / \partial x & \partial / \partial y & \partial / \partial z \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla \rightarrow \frac{1}{L}$$

# Konvektif Türev

$$\frac{D\boldsymbol{v}}{Dt} \equiv \frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v}$$

$$\frac{\boldsymbol{v}}{t} \equiv \frac{\boldsymbol{v}}{t} + \left(\boldsymbol{v}\,\frac{1}{L}\right)\boldsymbol{v}$$

$$\frac{L/T}{T} \equiv \frac{L/T}{T} + \left(\frac{L/T}{L}\right)\frac{L}{T}$$

$$\frac{L}{T^2} \equiv \frac{L}{T^2} + \frac{L}{T^2}$$

# Manyetik İndüksiyon Eşitliği

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 B$$

$$\frac{B}{T} = \frac{1}{L}\frac{L}{T} B + \eta \frac{B}{L^2}$$

$$\frac{1}{T} = \frac{1}{T} + \frac{\eta}{L^2}$$

$$\frac{1}{T} = \frac{\eta}{L^2}$$

$$\eta = 8 \times 10^8 T^{-3/2} m^2 s^{-1} \quad \textit{Renkkürede}$$

$$\eta = 10^9 T^{-3/2} m^2 s^{-1} \quad \textit{Güneştacında}$$

# Maxwell Eşitlikleri Işık Hızının Boşluk Değerini Öngörür

$\varepsilon_0 = 8.854 \times 10^{-12}\, farad\, m^{-1}$

$\mu_0 = 4\pi \times 10^{-7}\, henry\, m^{-1}$

$$\nabla \times \boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{H}}{\partial t}$$

$$\nabla \times \boldsymbol{H} = \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$

$$\nabla \times \frac{\partial \boldsymbol{H}}{\partial t} = \varepsilon_0 \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

$$\nabla \times \nabla \times \boldsymbol{E} = -\mu_0 \nabla \times \frac{\partial \boldsymbol{H}}{\partial t}$$

$$\nabla \times \nabla \times \boldsymbol{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

$$\frac{1}{L^2} = \frac{\varepsilon_0 \mu_0}{T^2}$$

$$\frac{L^2}{T^2} = c^2 = \frac{1}{\varepsilon_0 \mu_0} \qquad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

# Alfven Dalgaları

$$\rho \frac{D\boldsymbol{v}}{Dt} \equiv \rho \left[ \frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} \right] = \frac{1}{\mu}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}$$

$$\rho \frac{v}{T} = \frac{1}{\mu}\frac{1}{L} B^2$$

$$\frac{L}{T} v = v^2 = v_A^2 = \frac{B^2}{\mu \rho}$$

# Genel Dalga Denklemi

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2}$$

$$\rho \frac{D\boldsymbol{v}}{Dt} \equiv \rho\left[\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right] = \frac{1}{\mu}(\nabla\times\boldsymbol{B})\times\boldsymbol{B}$$

$$\frac{V}{T} = \frac{1}{L}\frac{B^2}{\mu\rho}$$

$$\frac{L}{T^2} = V_A^2\,\frac{1}{L}$$

$$\frac{1}{T^2} = V_A^2\,\frac{1}{L^2}$$

$$\frac{\phi}{T^2} = V_A^2\,\frac{\phi}{L^2}$$

$$\frac{\partial^2 \phi}{\partial t^2} = V_A^2\,\frac{\partial^2 \phi}{\partial x^2}$$

# Ses Dalgaları

$$\rho \frac{D\boldsymbol{v}}{Dt} = -\nabla p$$

$$\frac{M}{L^3}\frac{v}{T} = \frac{1}{L}\frac{M}{LT^2}$$

$$\frac{v^2}{L^2} = \frac{1}{T^2}$$

$$\frac{1}{L^2} = \frac{1}{v^2}\frac{1}{T^2} \quad \Rightarrow \quad \frac{\phi}{L^2} = \frac{1}{v^2}\frac{\phi}{T^2} \quad \Rightarrow \quad \frac{\partial^2\phi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}$$

$$\frac{\partial^2\phi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}$$

# Manyetikses Dalgaları

$$\rho \frac{D\boldsymbol{v}}{Dt} \equiv \rho\left[\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right] = -\nabla p + \frac{1}{\mu}(\nabla\times\boldsymbol{B})\times\boldsymbol{B}$$

$$v_{ms} = \sqrt{c_s^2 + V_A^2}$$

## $qB/m$ Niceliğinin Fiziksel Boyutu?

$$m\frac{d\boldsymbol{v}}{dt} = q\left(\boldsymbol{v}\times\boldsymbol{B}\right)$$

# $f$ Niceliğinin Fiziksel Boyutu?

$$Dim(q) \to M^{1/2} L^{3/2} / T$$

$$c^2 k^2 = \omega^2 + \frac{q_p^2 \pi}{m \varepsilon_0} \int_{-\infty}^{+\infty} d v_{//} \int_0^{\infty} \frac{(\omega - k v_{//})(\partial f_0 / \partial v_\perp) + k v_\perp (\partial f_0 / \partial v_{//})}{\omega - k v_{//} \pm \omega_c} v_\perp^2 \, d v_\perp$$

Dim(q) → M^1/2 L^3/2 / T

$k = 2\pi/\lambda$

$\omega$→ frekans

$\varepsilon_0$ → Boyutsuz

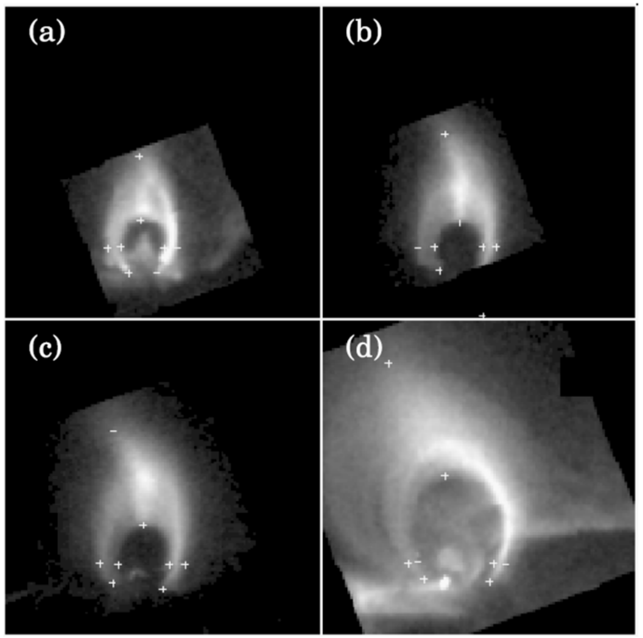
c – ışık hızı

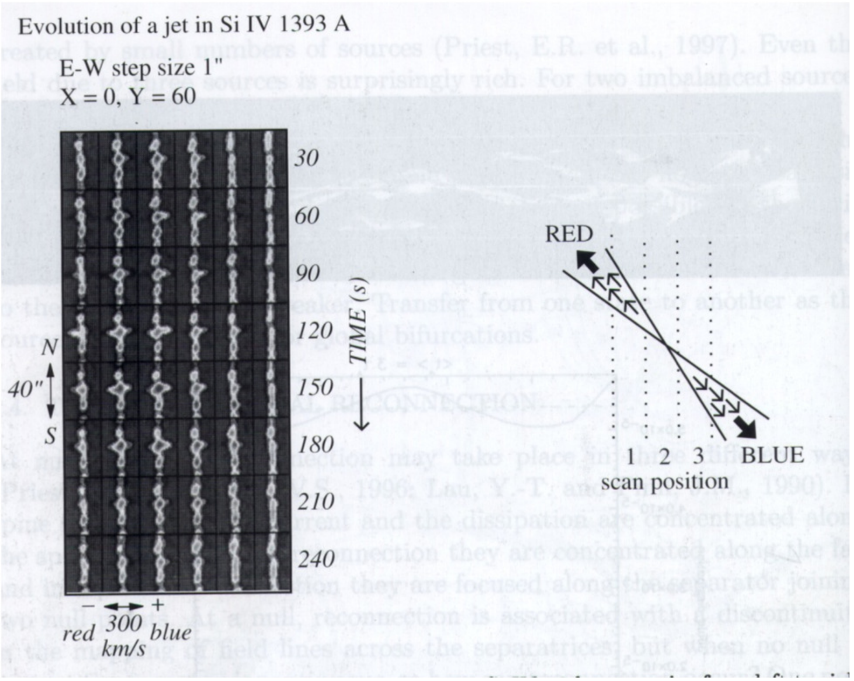
v – parçacık hızı

# Manyetik Reynolds Sayısı

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 B$$

$$\mathrm{Re}_m = \frac{|\nabla \times (\boldsymbol{v} \times \boldsymbol{B})|}{|\eta \nabla^2 B|} = \frac{\frac{1}{L} vB}{\eta \frac{B}{L^2}} = \frac{vL}{\eta}$$





# Plazma $\beta$ Parametresi

$$\frac{1}{\mu}\boldsymbol{J}\times\boldsymbol{B}=\frac{1}{\mu}(\nabla\times\boldsymbol{B})\times\boldsymbol{B}=\frac{1}{\mu}\left[(\boldsymbol{B}\cdot\nabla)\boldsymbol{B}-\frac{1}{2}\nabla B^{2}\right]$$

$$\rho\frac{D\boldsymbol{v}}{Dt}\equiv\rho\left[\frac{\partial\boldsymbol{v}}{\partial t}+(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right]=-\nabla p+\frac{1}{\mu}(\nabla\times\boldsymbol{B})\times\boldsymbol{B}=-\nabla p+\frac{1}{\mu}\left[(\boldsymbol{B}\cdot\nabla)\boldsymbol{B}-\frac{1}{2}\nabla B^{2}\right]$$

$$\beta=\frac{p}{B^{2}/2\mu}$$

# Reynolds Sayısı

$$\rho \frac{D\boldsymbol{v}}{Dt} \equiv \rho\left[\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right] = -\nabla p + \frac{1}{\mu}(\nabla\times\boldsymbol{B})\times\boldsymbol{B} + \rho\nu\left[\nabla^2\boldsymbol{v} + \frac{1}{3}\nabla(\nabla\cdot\boldsymbol{v})\right]$$

$$\mathrm{Re} = \frac{LV}{\nu}$$

# Rossby Sayısı

$$\rho \frac{D\boldsymbol{v}}{Dt} \equiv \rho\left[\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right] = -\nabla p + \frac{1}{\mu}(\nabla\times\boldsymbol{B})\times\boldsymbol{B} + \rho\nu\left[\nabla^2\boldsymbol{v} + \frac{1}{3}\nabla(\nabla\cdot\boldsymbol{v})\right] - 2\rho\boldsymbol{\Omega}\times\boldsymbol{v}$$

$$Ro = \frac{V}{L\Omega}$$

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# “Başka” Sayılar!

$$P_m = \frac{R_m}{\mathrm{Re}} = \frac{\nu}{\eta}$$ *Manyetik Prandtl sayısı*

$$M = \frac{V_0}{c_s}$$ *Mach Sayısı*

$$M_A = \frac{V_0}{V_A}$$ *Alfven Mach Sayısı*

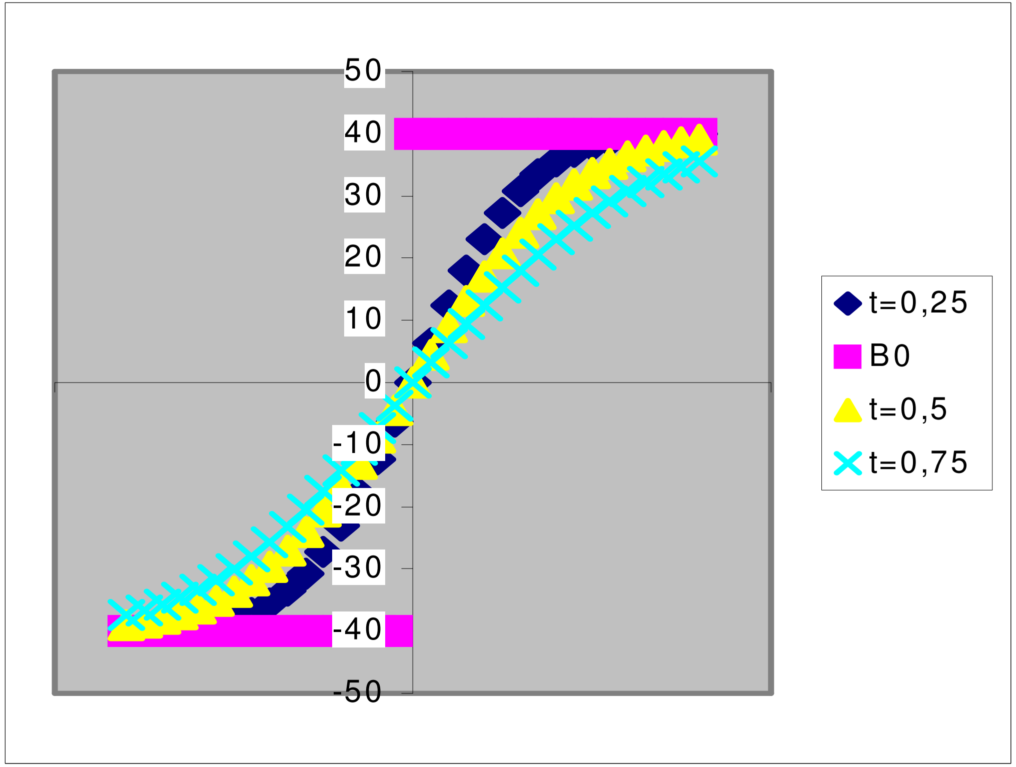
$$E = \frac{Ro}{\mathrm{Re}} = \frac{V}{l_0^2 \Omega}$$ *Ekman Sayısı*

$$T = E^{-2}$$ *Taylor Sayısı*

# Manyetik Difüzyon Zaman Ölçeği

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 B$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \eta \nabla^2 B$$



$$\frac{B}{T} = \eta \frac{B}{L^2}$$

$$\frac{1}{T} = \eta \frac{1}{L^2}$$

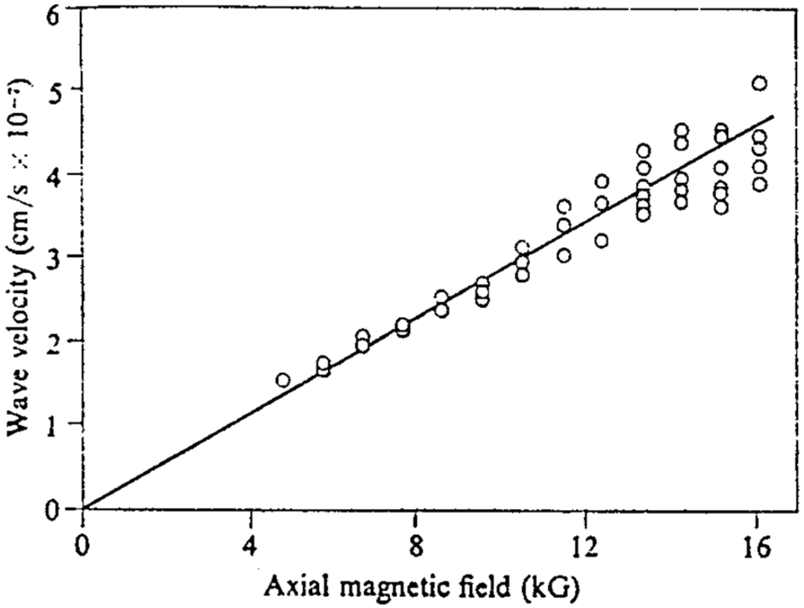
$$T = \frac{L^2}{\eta}$$

# Alfven Hızıyla Manyetik Alan Yeğinliği Arasındaki İlişki?

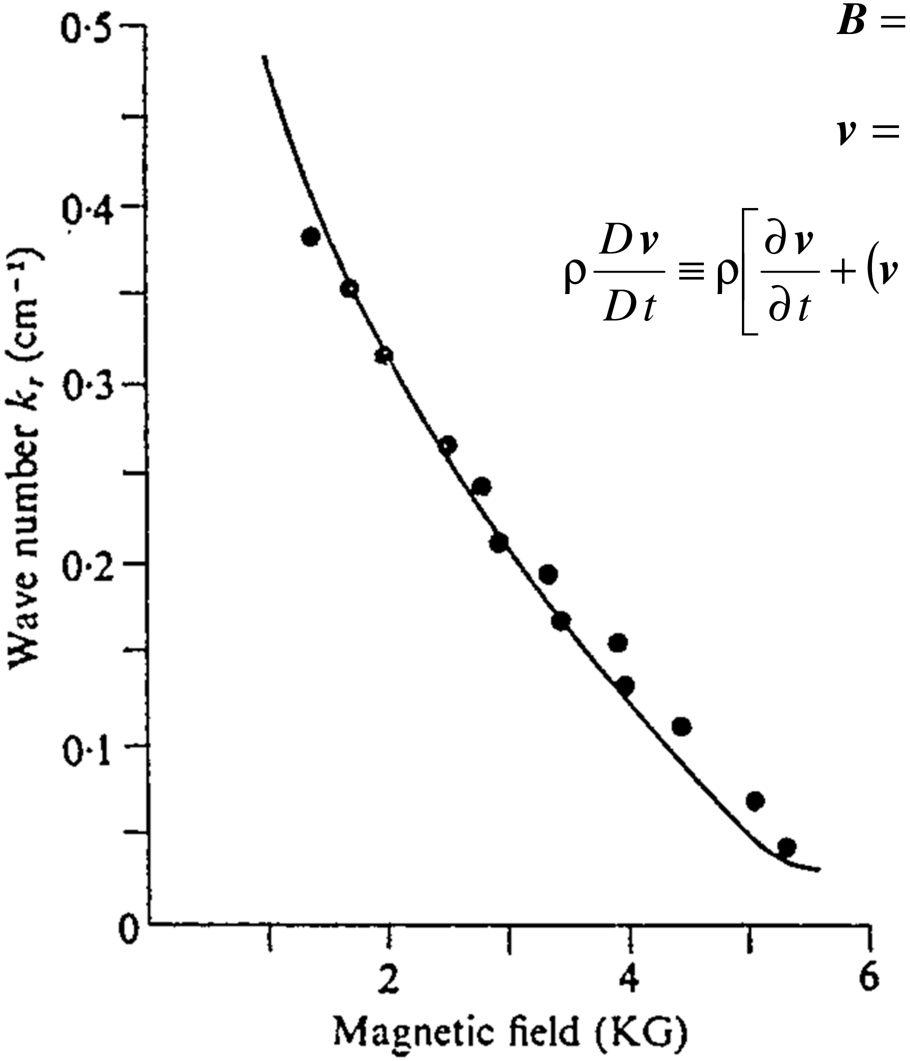
$$\rho\left[\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\nabla\right]\boldsymbol{v}=\left(\nabla\times\boldsymbol{B}\right)\times\frac{\boldsymbol{B}}{\mu}$$

$$\rho\frac{V}{T}=\frac{1}{L}\frac{B^2}{\mu}$$

$$V_A=\frac{B}{\sqrt{\mu\rho}}$$



# Alfven Dalgaları Dalga Sayısıyla Manyetik Alan Yeğinliği Arasındaki İlişki?



$$\boldsymbol{B} = \boldsymbol{B}_0 \exp\left[i\left(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t\right)\right]$$

$$\boldsymbol{v} = \boldsymbol{v}_0 \exp\left[i\left(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t\right)\right]$$

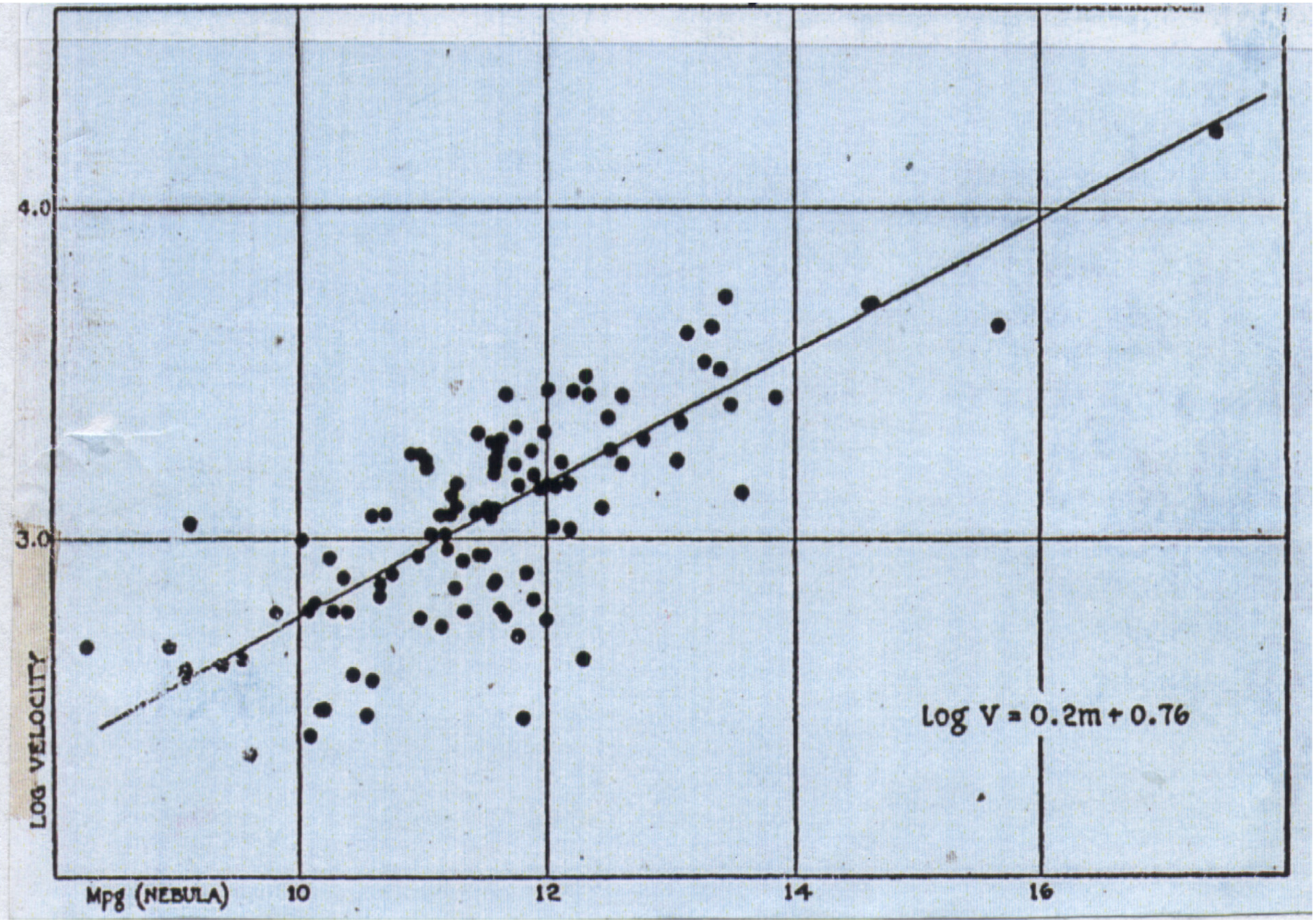
$$\rho \frac{D\boldsymbol{v}}{Dt} \equiv \rho\left[\frac{\partial \boldsymbol{v}}{\partial t} + \left(\boldsymbol{v}\cdot\nabla\right)\boldsymbol{v}\right] = \frac{1}{\mu}\left(\nabla\times\boldsymbol{B}\right)\times\boldsymbol{B}$$

$$-\rho i \omega = i\boldsymbol{k}\times\boldsymbol{B}\times\frac{\boldsymbol{B}}{\mu}$$

$$\rho\omega = k\frac{B^2}{\mu}$$

$$k = \frac{\rho\omega\mu}{B^2}$$

# Evrenin Genişlediğine İlişkin Görüş Hangi Gözlemsel Gerçekten Türetiliyor?



Insofar as it is possible to consider these forces in isolation, and if we make the assumption that gravity and pressure gradient forces are approximately equivalent, can the ponderomotive force account for the plasma accelerations observed? In a regime in which the Alfvén wave frequency $\omega \ll \omega_{ci}$ (which is almost certainly the case – see Haerendel, 1992), we may approximate Equation (14) by

$$\frac{\partial v_z}{\partial t} \approx \left(\frac{q_e}{m_p \omega_{ci}}\right)^2 \left(\partial_z + \frac{2k}{\omega}\partial_t\right) |E|^2. \tag{15}$$

Approximating $\partial_t \sim \omega$, and since the wavenumber $k$ is related to the frequency $\omega$ by $k \approx \omega / v_{\mathrm{A}}$, with the Alfvén speed $v_{\mathrm{A}} \equiv B/\sqrt{\mu_0 \rho} \sim 200$ km s$^{-1}$ for a region of strong magnetic field $B_z \approx 10^{-3}$ T, and density of $10^{10}$ cm$^{-3}$, we have $k \sim 3 \times 10^{-5}$ m$^{-1}$ for $\omega \sim 1$ Hz. Eliminating $\omega_{ci}$ in favor of $q_i$, $B_0$, and $m_i$ we find that

$$\frac{\partial v_z}{\partial t} \sim \left(\frac{E}{B_0}\right)^2 \left(\frac{1}{\Delta} + \frac{2\omega}{v_{\mathrm{A}}}\right), \tag{16}$$

where $\Delta$ is a length scale associated with the parallel wavelength of the wave. We estimate $\Delta \approx 10\lambda_z$, with $\lambda_z \approx 200$ km and so in order to achieve accelerations of the order of 1 km s$^{-2}$ we require an Alfvén wave with a field strength of order $E \approx 8$ V m$^{-1}$.

$$v = \frac{\mathbf{E} \times \mathbf{B}}{B^2} - \frac{\boldsymbol{\nabla} P \times \mathbf{B}}{qnB^2} = \mathbf{v}_E + \mathbf{v}_D,$$

$v_D$ is the diamagnetic drift, where **E** is a radial electric field, **B** is the magnetic field, $P$ is the plasma pressure $= nKT$, $q$ is the particle charge and $n$ is the plasma density;

$$\mathbf{v}_D = -\frac{\boldsymbol{\nabla} P \times \mathbf{B}}{qnB^2}.$$

For constant temperature and a density profile

$$v_D = \frac{KT \text{ (eV)}}{B \text{ (T)}} \frac{1}{L_n} \text{ m s}^{-1},$$

where $L_n$ is the density scale length in m for a 25 eV plasma.

Fizikteki eşitliklerde ortaya çıkan bazı parametrelerin evrenselliği ve doğasını açıklama çabaları vardır. Örneğin,
Newton fiziğindeki çekim sabiti, G; Planck sabiti, h; elektron yükü, q.
Diğer yandan bu sabitlerin evrenin yaşıyla birlikte değişip değişmediği de araştırılıyor.
Ölçümlerdeki küçük belirsizlikleri de dikkate alırsak, bu sabitlerin değiştiğine ilişkin çalışmaların başarısızlıkla sonuçlandığı rapor ediliyor (Wesson, 2001).
Temel sabitler adı verilen bu niceliklere parametre gözüyle de bakılabilir; bu sabitler diğer niceliklerin fiziksel boyutlarını geometrinin ele alabileceği türe dönüştürebilir.
Kısacası, $\left(c^2/G\rho\right)^{1/2} = [L]$ akışkanın yoğunluğunu uzunluğa dönüştürür.
Benzer biçimde, $h/mc = [L]$ parçacığın durgun kütlesini yine uzunluğa dönüştürür.
Bu yöntem, Genel Görelilikte olduğu gibi, fiziksel niceliklerin geometrik niceliklerle ilişkisini kurar.